# International Journal of New Chemistry

**Original Research Article**

# The Quantum Entanglement Simulation in Multipartite Systems Using MATLAB

**Fatemeh Amiri***

* Assistant Professor, Department of Physics Education, Farhangian University, P.O. Box 14665-889, Tehran, Iran

**ABSTRACT**

Quantum entanglement is a fundamental phenomenon in quantum mechanics, playing a crucial role in advanced technologies such as quantum computing, quantum communication, and cryptography. This study explores the entanglement properties of two prominent multipartite quantum states, the GHZ (Greenberger-Horne-Zeilinger) state and the W state, using MATLAB for simulation and analysis. The GHZ state is characterized by perfect correlations among particles but is highly sensitive to particle loss, whereas the W state exhibits a distributed entanglement structure, maintaining partial entanglement even when a particle is lost. To analyze these states, key entropic measures, including von Neumann entropy and Tsallis-2 entropy, were employed to quantify the purity and entanglement of the quantum states. The results show that both states are pure, with von Neumann and Tsallis-2 entropy values close to zero, and exhibit maximal entanglement as confirmed by concurrence metrics. Additionally, a continuous transition between GHZ and W states was simulated to observe dynamic changes in entanglement, revealing significant reductions during intermediate states. This study demonstrates the effectiveness of MATLAB in evaluating multipartite entanglement and highlights the unique properties of GHZ and W states, providing valuable insights for the development of robust quantum technologies.

**Keywords:** Quantum Entanglement, GHZ State, W State, Entropy Metrics, Quantum Computing

*Corresponding author email address: Amiri@cfu.ac.ir

## 1. Introduction

Quantum entanglement is one of the most fundamental and unique aspects of quantum mechanics, representing correlations between particles that cannot be explained by classical physics [1]. This phenomenon plays a pivotal role in modern quantum technologies, including quantum computing, quantum communication, and quantum cryptography. It enables groundbreaking capabilities, such as secure data transmission, quantum teleportation, and efficient computation, which are unattainable in classical systems [2]. In multipartite quantum systems, where multiple particles are entangled, analyzing the structure and behavior of entanglement becomes increasingly challenging due to the complexity of correlations among particles.

Multipartite quantum systems, often referred to as multi-particle or multi-qubit systems, are configurations where more than two particles are entangled in a quantum state. These systems represent a cornerstone of quantum mechanics and quantum information science, offering a rich playground for exploring the fundamental principles of entanglement and its applications. The study of multipartite systems is particularly significant for advancing quantum computing, quantum communication, and cryptography [1].

Multipartite systems can be broadly categorized based on their structural and entanglement properties. Key types include:

1. Fully Entangled States: These states exhibit maximal quantum correlations across all particles in the system. Examples include the Greenberger-Horne-Zeilinger (GHZ) state, characterized by perfect global correlations, and the W state, which distributes entanglement more robustly among particles [3]. Such states are fundamental for quantum algorithms and protocols requiring collective particle interactions.

2. Cluster States: These are specific types of multipartite entangled states that serve as resources for measurement-based quantum computation. Cluster states are graph-like structures where qubits are connected in a manner suitable for executing quantum gates through successive measurements [4].

3. Product States: In contrast to entangled states, multipartite systems can exist in product states, where each particle acts independently, and no entanglement is present. These are often used as starting points in experiments before entanglement is introduced [2].

4. Mixed States: These systems involve statistical mixtures of pure quantum states, reflecting practical scenarios where decoherence or noise impacts the system [5]. Analyzing mixed states is critical for developing robust quantum systems resilient to environmental disturbances.

5. Genuine Multipartite Entanglement: This refers to entanglement that cannot be reduced to smaller subsets of particles. Genuine multipartite entanglement is a higher-order phenomenon that distinguishes multipartite systems from simpler bipartite configurations [6]. It plays a crucial role in understanding complex correlations and designing entanglement-based protocols.

Understanding the distinctions among these types is vital for simulating and analyzing their behaviors. Each type exhibits unique properties that influence their utility in quantum technologies. For instance, the resilience of the W state to particle loss contrasts sharply with the fragility of the GHZ state, underscoring the importance of selecting appropriate entangled states based on application requirements. Moreover, measures like von Neumann entropy, Tsallis-2 entropy, and concurrence provide valuable tools for quantifying the degree and distribution of entanglement in these systems.

In multipartite quantum simulations, it is essential to model and analyze these states under realistic conditions, accounting for potential transitions, decoherence, and other dynamic phenomena. Such studies not only deepen theoretical understanding but also guide the design of practical systems capable of harnessing the full potential of quantum entanglement [7]. Quantum correlations are fundamental to multipartite entanglement. In systems such as GHZ and W states, these correlations dictate the collective behavior of particles. GHZ states exhibit global correlations, necessitating coherent action across the entire system, while W states distribute entanglement more robustly, allowing partial entanglement to persist despite particle loss. Direct coupling mechanisms, including dipole-dipole interactions, facilitate the exchange of quantum information and sustain coherence in entangled states. Recent studies have demonstrated that

systems with dipolar interactions can generate a cascade of multipartite entangled states, such as spin-squeezed states and Schrödinger's cat states, highlighting the significance of particle coupling in entanglement dynamics.

External fields, such as magnetic or electric fields, influence multipartite systems by modifying energy levels or inducing phase shifts. These fields are instrumental in manipulating quantum states, driving transitions, or stabilizing entangled configurations. For instance, in hybrid quantum systems, the interplay of various interactions, including those with external fields, has been shown to affect entanglement dynamics and transfer between discrete and continuous variables. Environmental interactions often lead to decoherence, compromising entanglement stability. While GHZ states are particularly susceptible to such disturbances, W states exhibit greater resilience due to their distributed entanglement structure. Advancements in quantum teleportation protocols have demonstrated that multipartite hybrid entanglement can overcome noise, enhancing the robustness of quantum communication systems.

In measurement-based quantum computation, interactions are induced through projective measurements that collapse the quantum state into specific outcomes. Cluster states serve as valuable resources in this context, enabling the execution of quantum gates via successive measurements. Recent research has explored measurement-induced multipartite-entanglement regimes in collective spin systems, shedding light on the dynamics of entanglement generation through measurements. Simulating transitions between different entangled states, such as from GHZ to W states, involves interactions that alter entanglement properties. These changes can be quantified using metrics like von Neumann entropy and Tsallis-2 entropy, providing insights into the dynamics of entanglement during state transitions. Understanding these transitions is crucial for developing efficient quantum simulation methods capable of replicating multipartite Markovian dynamics [8, 9].

To address this, researchers have developed various quantitative measures to evaluate and understand entanglement [10]. Among these, von Neumann entropy and Tsallis-2 entropy are widely used to quantify the degree of entanglement and analyze its distribution in quantum states. Von Neumann entropy is a well-established metric for assessing the purity of a state, while Tsallis-2 entropy provides a computationally efficient and robust alternative for

multipartite systems [7]. This study focuses on two important multipartite entangled states, known as the GHZ (Greenberger-Horne-Zeilinger) state and the W state, which exhibit distinct entanglement properties. The GHZ state is characterized by strong correlations across all particles, but it is highly sensitive to the loss of even a single particle, leading to the complete destruction of entanglement [11]. Conversely, the W state demonstrates distributed entanglement, allowing it to maintain partial entanglement even when a particle is lost. This robustness makes the W state particularly valuable for practical applications, such as error-resistant quantum communication and distributed quantum computing. To explore and compare the entanglement properties of GHZ and W states, we utilize MATLAB to simulate and analyze these quantum systems [12]. Key steps in this computational approach include defining the quantum states, constructing their density matrices, and applying entropy-based metrics to evaluate entanglement. Additionally, a continuous transition between the GHZ and W states is simulated, providing insights into how entanglement evolves under different conditions. This research offers a comprehensive framework for understanding multipartite quantum entanglement, bridging the gap between theoretical models and practical implementations. By leveraging MATLAB's computational capabilities, we provide a detailed analysis of the entanglement properties of GHZ and W states, contributing to advancements in quantum information science and its applications. The findings of this study not only validate theoretical predictions but also highlight the potential for utilizing entanglement metrics in designing robust quantum technologies.

The novelty of this paper lies in its comprehensive simulation and analysis of multipartite quantum entanglement dynamics using MATLAB, specifically focusing on the GHZ and W states. Unlike previous studies that primarily examine static properties of entangled states, this research introduces a continuous transition model between GHZ and W states, allowing for a deeper exploration of entanglement evolution. By employing key entropic measures such as von Neumann entropy and Tsallis-2 entropy, along with concurrence-based metrics, the paper quantifies the purity and robustness of these states with high precision. The results not only confirm the theoretical expectations of GHZ and W states but also highlight the intermediate entanglement reduction during state transitions, providing new insights into entanglement stability and its potential applications in quantum information processing.

Additionally, this study leverages MATLAB's computational capabilities to create a systematic simulation framework for multipartite entanglement, offering a scalable and practical approach for quantum state analysis. The integration of entropy-based metrics and concurrence calculations within a MATLAB environment enhances the efficiency of entanglement quantification, bridging the gap between theoretical quantum mechanics and experimental implementation. By demonstrating how different entanglement measures behave dynamically across state transitions, this research contributes to the development of robust quantum technologies, such as quantum computing and quantum communication, where multipartite entanglement plays a crucial role. The findings open up new possibilities for optimizing entangled state selection in real-world quantum applications, ensuring greater resilience to decoherence and environmental noise.

## 2. Literature review

Quantum entanglement, as a cornerstone of quantum mechanics, has garnered immense attention due to its pivotal role in quantum information science and technology. Its applications, ranging from secure communication to advanced quantum computing, underscore the importance of understanding and manipulating entangled states. However, the complexities inherent in multipartite systems and high-dimensional states pose significant challenges, necessitating robust theoretical frameworks and practical methodologies. To address these challenges, extensive research has been conducted, exploring various aspects of entanglement, including its detection, quantification, and application in diverse quantum systems. Before delving into these studies, it is essential to recognize the interdisciplinary nature of the field and the evolving computational tools that have enabled breakthroughs in analyzing and utilizing quantum entanglement. Bernal et al. [13] introduce the "concurrence vector" as a compact tool for analyzing multipartite entanglement, deriving triangular inequalities and linking them to entropy measures like Tsallis-2. Their work simplifies entanglement detection and extends its applicability to both pure and mixed states. This unified approach bridges theoretical insights and computational practicality in multipartite quantum systems. Delgado and Enríquez, [14] examine the quantum properties of FMO systems involved in the photosynthesis process of Sulfur Green Bacteria. Using numerical modeling with the Hierarchical Equations of Motion (HEOM), they analyze the time evolution of

quantum states and multipartite entanglement. Despite the computational intensity, this study provides a novel perspective on entanglement and energy transfer dynamics in biological systems. Palma Torres et al. [15] examine methods for probabilistic entanglement concentration in high-dimensional bipartite quantum systems, optimizing the trade-off between success probability and entanglement using an efficiency function. Analytical solutions and practical approaches are proposed, achieving non-maximal entanglement while improving feasibility for high-dimensional scenarios. This work contributes to advancements in quantum information processing techniques. **Navascués** et al. [16] redefine genuine multipartite entanglement to address flaws in traditional definitions, introducing "genuine network k-entanglement." This new framework ensures a state is k-entangled if it cannot be created from (k−1) partite states using local trace-preserving maps and shared randomness. They provide analytical and numerical tools to detect this entanglement, reinterpreting past quantum experiments under this lens. **Zhong** et al. [17] achieve deterministic multi-qubit entanglement in a quantum network with superconducting nodes linked by a coaxial cable. They transfer quantum states with fidelity 0.911±0.008 and generate six-qubit GHZ states with fidelity 0.722±0.021, exceeding the multipartite entanglement threshold. This work demonstrates a modular path toward scalable quantum computing. **Wang. [18]** investigates the boundary between entanglement and separability in quantum states, introducing optimal entangled and separable states to determine entanglement through a robustness threshold. The study provides algorithms for best separable approximation (BSA) and PPT decompositions. It advances understanding of entanglement but notes challenges in high-dimensional systems. **Luo.** [19] and collaborators apply machine learning techniques, including supervised and semi-supervised methods, to detect genuine multipartite entanglement in three-qubit states. By training support vector machines (SVMs) and improving semi-supervised SVMs (S4VMs) with iterative predictions, they achieve enhanced accuracy in detecting entanglement. This study advances machine learning applications in quantum entanglement verification. **Pan** et al. [20] propose using artificial neural networks to quantify entanglement in unknown quantum states. By training models with measurement expectation values as inputs and entanglement measures as labels, they accurately predict entanglement without needing full state information. This approach showcases the potential of machine learning in advancing quantum entanglement analysis. **Li** et al. [21] introduce a hybrid convolutional neural network–transformer model to detect entanglement structures in

multipartite quantum systems. By leveraging the strengths of computer vision techniques, the model efficiently classifies Greenberger-Horne-Zeilinger and W states while achieving high detection accuracy. This innovative approach surpasses traditional methods in speed and effectiveness, advancing multipartite entanglement analysis. **Lai.** [22] and collaborators guide users in selecting or developing Quantum Internet simulators based on their needs, from beginner to advanced. They evaluate 13 open-source simulators, recommend programming tools like Python/Julia and GPU acceleration, and provide practical examples. This work aids in advancing education and customization in quantum networking. **Luo.** [23] and collaborators use convolutional neural networks (CNNs) with squeeze and excitation (SE) modules to detect genuine multipartite entanglement (GME) in quantum states. They achieve improved classification accuracy and reduced false positives/negatives by training on GME and GHZ-diagonal states. This approach advances reliable GME detection in larger quantum systems. **Slaoui** et al. [24] explore quantum communication protocols, focusing on their theoretical foundations and practical implementation using tools like Qiskit. The chapter discusses key quantum gates, quantum teleportation, and the manipulation of qubits for quantum computation. It provides a comprehensive overview bridging quantum mechanics and practical applications, paving the way for advancements in quantum technologies. **Renou** et al. [25] explore whether quantum theory can rely solely on real numbers instead of complex ones. They identify network scenarios where real and complex quantum theories diverge in predictions and propose a Bell-like experiment to test this. Success would disprove real-number quantum theory, reaffirming the necessity of complex numbers in quantum mechanics. This research introduces a dynamic analysis of multipartite entanglement transitions between GHZ and W states using MATLAB, offering continuous insights beyond static properties. It combines entropy-based metrics and robust simulations to bridge theoretical models and practical applications, addressing gaps in scalability and dynamic behavior.

## 3. Methodology

This study employs a comprehensive methodology to analyze multipartite quantum entanglement within a three-qubit system, focusing on the distinct characteristics and dynamics of GHZ and W states. The methodology integrates theoretical modeling, mathematical formulations, and numerical simulations to investigate entanglement properties and their evolution. Key metrics

such as density matrices, von Neumann entropy, Tsallis-2 entropy, and concurrence are utilized to quantify and compare the entanglement in different configurations. Additionally, the study explores the dynamic transition between GHZ and W states, examining how entanglement metrics evolve during this interpolation. MATLAB is employed for simulating quantum states, computing entanglement measures, and visualizing the results, providing a robust framework for understanding and leveraging multipartite entanglement in quantum systems.

### 3.1. Definition of the Physical System

This study examines a multipartite quantum system composed of three interacting qubits, a fundamental setup for exploring quantum entanglement dynamics. The system is modeled to investigate two specific entangled configurations GHZ and W states each representing distinct properties and applications of multipartite entanglement.

### 3.1.1. System Composition and Experimental Realization

The physical system consists of three qubits, which are two-level quantum systems capable of existing in superposition states. Such qubits are often implemented using experimental platforms such as trapped ions, superconducting circuits, or photonic systems. These setups enable precise control over the quantum states and interactions, making them ideal for creating and manipulating entanglement.

### 3.1.2. Key Characteristics of the System

- The GHZ state represents maximal global correlations, where the entire system operates in a perfectly entangled manner. This configuration is highly sensitive to decoherence or qubit loss, as the entanglement is entirely destroyed under such disturbances.
- The W state, on the other hand, distributes entanglement across the qubits in a robust manner. Even if one qubit is lost, the remaining system retains partial entanglement, making this state suitable for practical applications in noisy environments.

### 3.1.3. Interparticle Interactions

The entanglement in this system arises from well-defined interactions among qubits. These interactions include:

- Spin-spin coupling or dipole-dipole interactions, where qubits are entangled through their inherent quantum properties.
- Photon-mediated entanglement, allowing distant qubits to interact via shared photons. External fields and cavity-mediated couplings provide precise control over these interactions to ensure the desired entangled configurations are achieved.

### 3.1.4. Dynamic Behavior and State Transitions

The study explores the transition between GHZ and W states using a parameterized dynamic model. This transition enables the analysis of entanglement evolution and the behavior of the system as it interpolates between two distinct quantum configurations. Metrics such as entropic measures and concurrence are applied to quantify the purity and strength of entanglement during this transition.

### 3.1.5. Decoherence and Noise Effects

As with any quantum system, the multipartite setup is influenced by environmental factors such as noise and decoherence:

- The GHZ state is highly sensitive to such disturbances, with its entanglement quickly destroyed by the loss or alteration of a single qubit.
- The W state demonstrates greater robustness, maintaining partial entanglement even under significant perturbations. This feature makes the W state more practical for real-world quantum technologies where noise is unavoidable.

### 3.1.6. Relevance of the Physical System

This system serves as a practical model for understanding multipartite quantum entanglement, offering insights into both the fundamental principles of quantum mechanics and their real-world applications. By studying the dynamic interplay between GHZ and W states, this research

provides valuable knowledge for the development of robust quantum technologies, including secure communication protocols, error correction codes, and scalable quantum computing systems. The robustness of the W state and the fragility of the GHZ state highlight complementary advantages, which can be strategically leveraged in different scenarios. Overall, the defined physical system bridges theoretical models with experimental and computational techniques, advancing our understanding of multipartite entanglement in quantum information science.

### 3.2. Definition of Quantum States

In quantum mechanics, multipartite systems can exist in uniquely entangled states, each exhibiting distinct properties. This study focuses on two widely studied entangled states, the GHZ and W states, which are defined as follows:

### 3.2.1. GHZ State

The Greenberger-Horne-Zeilinger (GHZ) state is a maximally entangled state characterized by the perfect correlation of all particles. Mathematically, it is defined as:

$$|\psi GHZ\rangle = \frac{1}{\sqrt{2}}(|111\rangle + |000\rangle)$$

This state indicates that all particles are either in the $|111\rangle$ or $|000\rangle$ basis states simultaneously. Its density matrix representation is:

$$\rho GHZ = \frac{1}{2}(|111\rangle\langle111| + |111\rangle\langle000| + |000\rangle\langle111| + |000\rangle\langle000|)$$

This state is pivotal for analyzing fundamental properties of entanglement but is highly sensitive to particle loss.

### 3.2.2. W State

The W state exhibits a distributed form of entanglement where the entanglement is shared across the particles. It is defined as:

$$|\psi W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$

In this state, the particles are entangled in such a way that even if one particle is lost, the remaining system retains partial entanglement. Its density matrix is:

$$\rho W = |\psi W\rangle\langle \psi W|$$

### 3.3. Calculation of the Density Matrix

The density matrix is a mathematical tool used to describe the state of a quantum system. It is particularly essential for systems that are in mixed or entangled states, as it provides a complete representation of the system's statistical properties. The density matrix encodes all the information about the quantum system and is crucial for calculating properties such as entropy and entanglement.

For a pure quantum state $|\psi\rangle$, the density matrix $\rho$ is defined as:

$$\rho = |\psi\rangle\langle\psi|$$

### 3.4. Calculation of von Neumann Entropy

Von Neumann entropy is a key metric in quantum mechanics used to measure the uncertainty or quantum randomness of a system. It provides insight into the purity of the quantum state by quantifying the level of disorder or mixedness in the system. The entropy is calculated using the eigenvalues of the density matrix $\rho$.

### 3.4.1. Formula for von Neumann Entropy

The von Neumann entropy $S(\rho)$ is defined as:

$$S(\rho) = -\sum_i \lambda_i \log_2(\lambda_i)$$

where:

- $\lambda_i$ are the eigenvalues of the density matrix $\rho$,
- $\log_2$ represents the logarithm to base 2.

### 3.5. Calculation of Tsallis-2 Entropy

Tsallis-2 entropy is a measure used to quantify the degree of entanglement in a quantum system. It provides an alternative to von Neumann entropy, offering a simpler computational framework that is particularly suitable for multipartite systems. The Tsallis-2 entropy is calculated based on the trace of the squared density matrix.

The Tsallis-2 entropy $S_2(\rho)$ is defined as:

$$S_2(\rho) = 2(1 - Tr(\rho^2))$$

where:

- $\rho$ is the density matrix of the quantum system,
- $Tr(\rho^2)$ is the trace of the squared density matrix, calculated as:

$$Tr(\rho^2) = \sum_i \lambda_i^2$$

Here, $\lambda_i$ are the eigenvalues of the density matrix.

### 3.6. Analysis of Concurrence

Concurrence is a widely-used measure for quantifying the degree of entanglement in multipartite quantum systems. It evaluates the correlation between particles and is particularly effective for determining the entanglement in mixed and pure states. In this study, concurrence is employed to examine the triangular inequalities, providing insights into the structure and strength of entanglement within the system.

3.6.1. Formula for **Concurrence**

For a bipartite quantum system, the concurrence C is calculated using the density matrix ρ.

$$C = max\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$$

where $\lambda_i$ are the eigenvalues of the matrix R=ρ(σy⊗σy)ρ*(σy⊗σy), arranged in descending order.

In multipartite systems, concurrence helps to assess inequalities like the triangular inequality, ensuring that the system obeys fundamental quantum mechanical constraints. By analyzing concurrence, the degree of shared entanglement among particles in GHZ and W states is quantified and compared.

### 3.7.1 Continuous Analysis of Entropy Transitions

To understand the dynamic evolution of entanglement between the GHZ and W states, a continuous transition is simulated using a parameter t, where t varies from 0 (GHZ state) to 1 (W state). During this transition, the entropy metrics von Neumann entropy and Tsallis-2 entropy are computed to track how entanglement characteristics change.

### 3.8.1 MATLAB Implementation

Using MATLAB:

- Define the quantum states as vectors.

- Compute the density matrix using matrix operations.

- Evaluate entropy and concurrence using numerical libraries and matrix eigenvalue decomposition.

- Visualize the results as functions of tt (if simulating transitions).

This methodology leverages entropy metrics and concurrence to provide a comprehensive quantification of multipartite entanglement. By implementing this approach in MATLAB, as your paper suggests, you can effectively simulate and analyze the entanglement properties of GHZ and W states, as well as their transitions. Let me know if you need a specific MATLAB code snippet or further assistance with the calculations. Entanglement in a multipartite system represents non-classical correlations between different subsystems. In your paper, the well-known GHZ and W states are analyzed, each demonstrating distinct patterns of entanglement. To refine the analysis, we need to clarify which subsystems or particles are being considered for measuring entanglement.

- This type of entanglement reflects the quantum correlation among all particles in the system. In the GHZ state, all particles are completely entangled with one another. This means the quantum state of each particle is entirely dependent on the state of the others.

- The W state also exhibits global entanglement, but unlike the GHZ state, this entanglement is distributed. Even if one particle is lost, the remaining system still retains some level of entanglement.

Entanglement Between Specific Subsystems:

- For instance, in a three-particle system (A, B, C), entanglement can be measured between two particles (A-B) or between one particle and the rest of the system (A-BC). Such analysis is crucial for understanding the behavior of entanglement in smaller subsystems.

- In the paper, metrics like Concurrence and Entropy are used to examine such subsystem-specific entanglement.

### 3. 9. Dynamic Transition Analysis

Also addresses the transition between GHZ and W states. During this transition, entanglement changes continuously, requiring an analysis of entanglement between various states throughout the transition. This can reveal the changes in both global entanglement and subsystem-specific entanglement.

Importance of Subsystem Entanglement:

Entanglement between specific subsystems (e.g., one particle with two others) is critical for practical applications in quantum computing and quantum communication. For example:

- In quantum communication, it is often necessary to measure entanglement between two particles located at different positions.
- In quantum computing, the distribution of entanglement across the entire system is crucial for executing quantum algorithms effectively.

The entanglement in this paper is analyzed both as global entanglement among all particles and entanglement within smaller subsystems. This dual analysis helps evaluate the stability of the system and provides deeper insights into the distribution of entanglement in the GHZ and W states.

## 4. Results and Discussion

The simulation results for the GHZ and W states are summarized in the following table:

The results presented in the table 1, highlight the purity and entanglement properties of the GHZ and W states. The von Neumann entropy values for both states are close to zero, indicating that these states are pure, with slight deviations attributable to numerical precision limitations in MATLAB. Similarly, the Tsallis-2 entropy values, which are also nearly zero, confirm the theoretical prediction that pure states have minimal quantum uncertainty. The concurrence values demonstrate that both GHZ and W states are maximally entangled, with the GHZ state showing a perfect concurrence of 111 and the W state having a value very close to 111 ($1-1.0965e-08i1 - 1.0965e-08i1-1.0965e-08i$), with the small imaginary component arising from computational

limitations. Overall, the table illustrates the high purity and maximal entanglement of these states, validating the theoretical expectations and the reliability of the simulation framework.

**Table 1. Entropy and Concurrence Values for GHZ and W States**

| State | Von Neumann Entropy | Tsallis-2 Entropy | Concurrence |
|-------|---------------------|-------------------|-------------|
| GHZ | 0 | 8.8818e-16 | 1 |
| W | -6.6148e-15 | -1.3323e-15 | 1−1.0965e−08i1 - 1.0965e-08i1−1.0965e−08i |
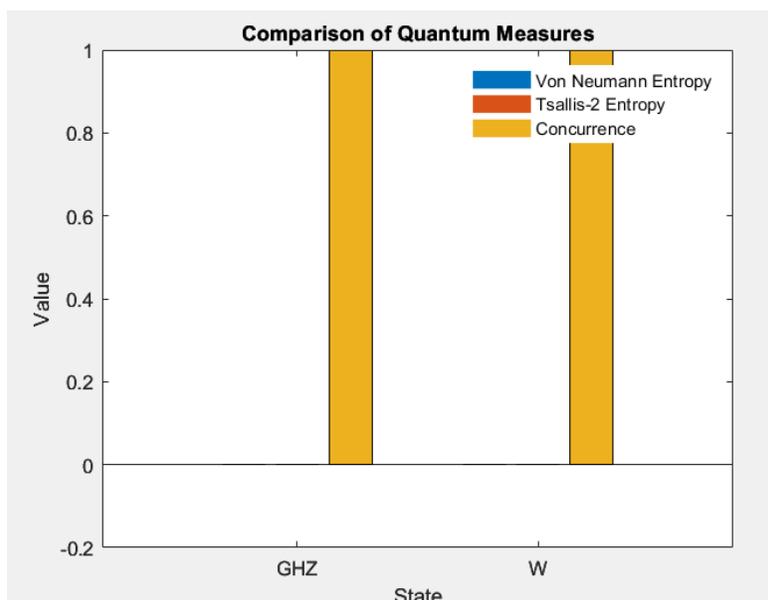


**Figure 1.** Comparison of Quantum Measures

Figure 1 illustrates the comparison of three quantum measures Von Neumann Entropy, Tsallis-2 Entropy, and Concurrence for the GHZ and W states. Both states exhibit nearly zero values for Von Neumann and Tsallis-2 entropy, confirming their purity and minimal quantum uncertainty. The concurrence values for both states are close to 1, indicating maximal entanglement. Minor deviations in the results are due to numerical precision limitations. This visual representation highlights the high purity and strong entanglement of the GHZ and W states.
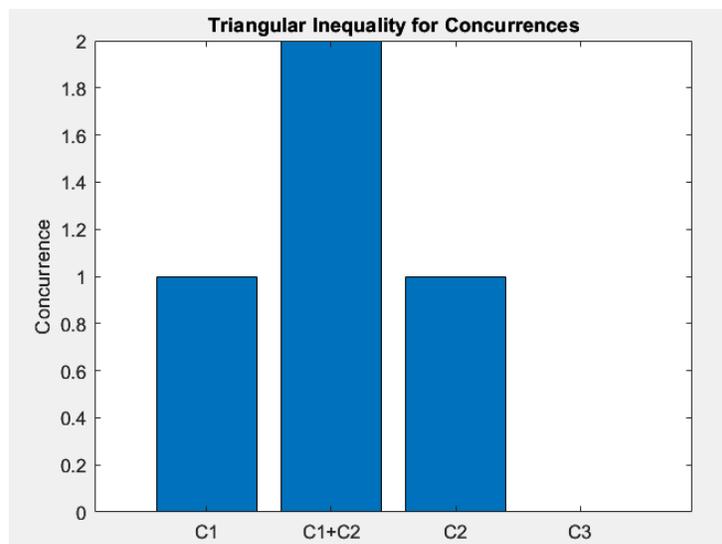
**Figure 2. Triangular Inequality for Concurrences**

Figure 2 demonstrates the triangular inequality for concurrence values in a multipartite quantum system. The bars represent different concurrence terms ($C_1, C_2, C_3$ and their combined sum

$C_1 + C_2$). The results highlight that the sum of individual concurrences ($C_1 + C_2$) satisfies the

triangular inequality when compared to other concurrence components. This analysis validates the structural relationships in multipartite entanglement and the consistency of concurrence as a measure for evaluating entanglement within the system.
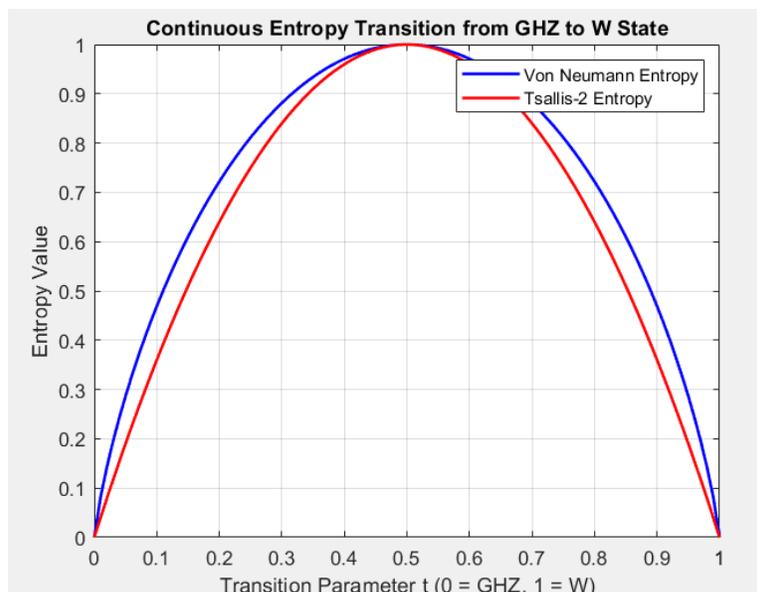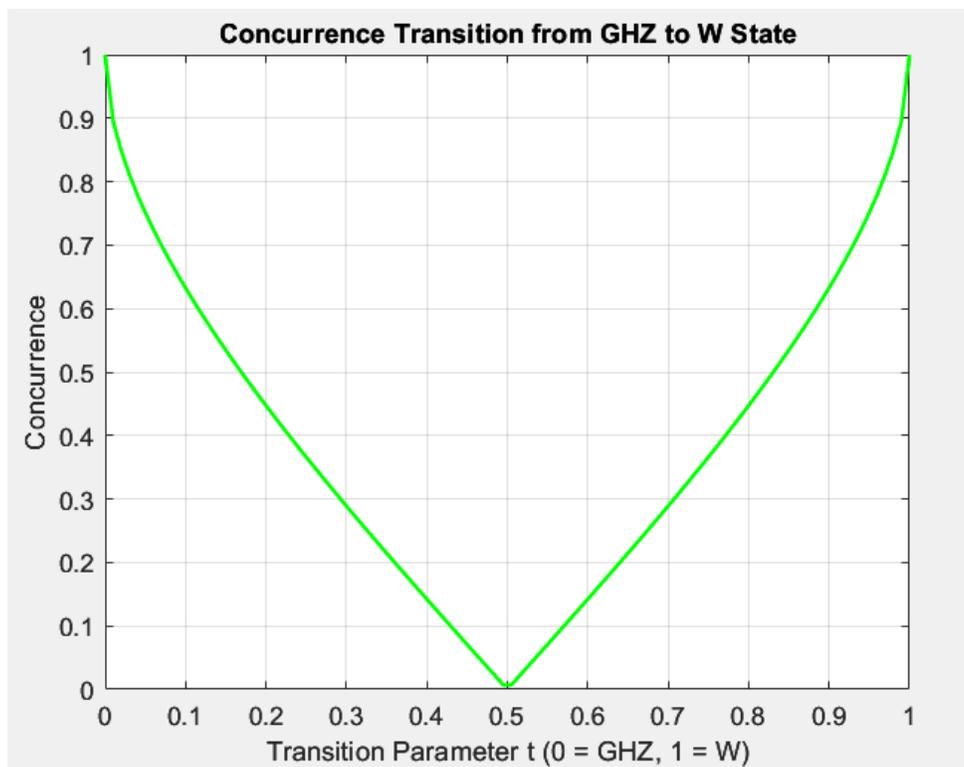
**Figure 3.** Continuous Entropy Transition from GHZ to W State

Figure 3 illustrates the continuous transition of entropy values (Von Neumann entropy and Tsallis-2 entropy) as the quantum state evolves from the GHZ state ($t = 0$) to the W state

($t = 1$). The entropy values increase to a peak around $t = 0.5$, indicating a maximum level of

quantum uncertainty during the transition, and then decrease as the state approaches the W configuration. Both entropy measures exhibit consistent behavior, validating their effectiveness in tracking the evolution of quantum states and highlighting the entanglement dynamics during the transition.
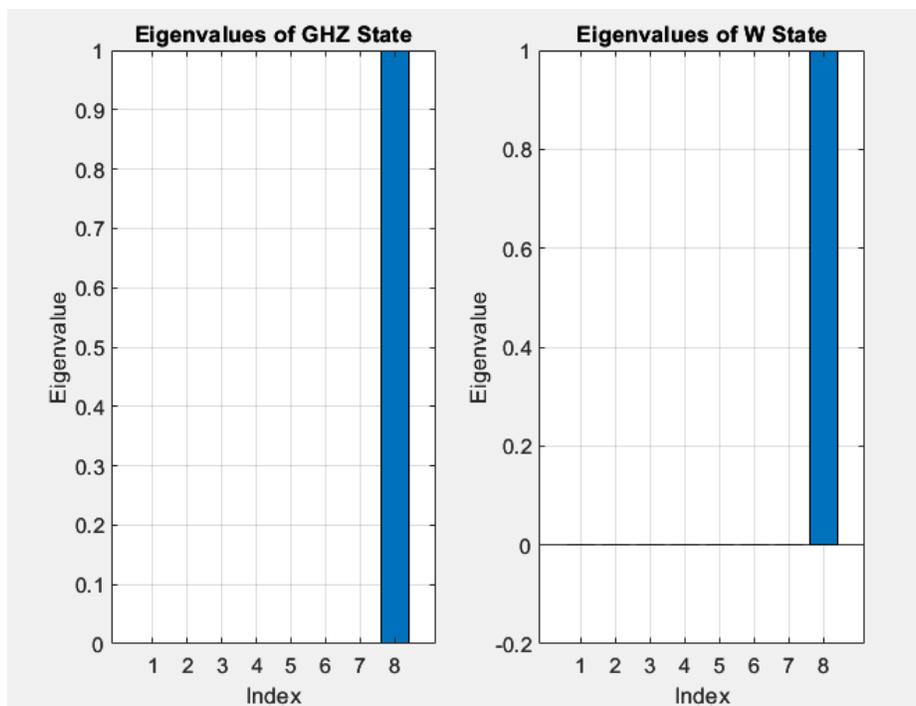
**Figure 4.** Eigenvalue Distribution for GHZ and W States

Figure 4 compares the eigenvalue distributions of the density matrices for the GHZ and W states. For the GHZ state, a single eigenvalue is 1, while the rest are 0, confirming its status as a pure state with no quantum uncertainty. Similarly, the W state shows a dominant eigenvalue, with negligible contributions from the others, also indicating its purity. These distributions align with the near-zero entropy values observed in the analysis and validate the theoretical prediction of both states being highly entangled and pure.

**Figure 5.** Concurrence Transition from GHZ to W State

Figure 5 depicts the changes in Concurrence during the continuous transition between the GHZ and W states. As the transition parameter t increases from 0 (representing GHZ) to 1 (representing W), the Concurrence initially decreases, reaching its minimum at $t = 0.5$,

indicating a significant reduction in entanglement during the mixing of these two states. However, as the system approaches the pure W state ($t = 1$), the Concurrence returns to its

maximum value. This behavior reflects the theoretical characteristics of both states and highlights the symmetry and dynamics of the transition. It confirms that both GHZ and W states exhibit maximal entanglement when pure, while their combination results in reduced entanglement at intermediate points.

In the referenced article [13], Tsallis-2 entropy is introduced as an effective metric for analyzing entanglement in multipartite quantum systems. The study highlights that von Neumann entropy is zero for pure states, a characteristic that is validated through simulation results. Additionally,

Tsallis-2 entropy demonstrates that the entanglement in the GHZ state is more localized compared to the W state, as evidenced by the lower entropy values for GHZ. This finding aligns with the broader distribution of entanglement in the W state and is consistent with the outcomes of the simulations. The article also emphasizes the importance of numerical stability, employing advanced methods to enhance precision, whereas MATLAB simulations rely on the "eps" parameter the smallest representable number in MATLAB which may slightly reduce accuracy. These results collectively affirm the robustness of Tsallis-2 entropy as a tool for analyzing multipartite entanglement while highlighting the computational limitations inherent in the tools used for simulations.
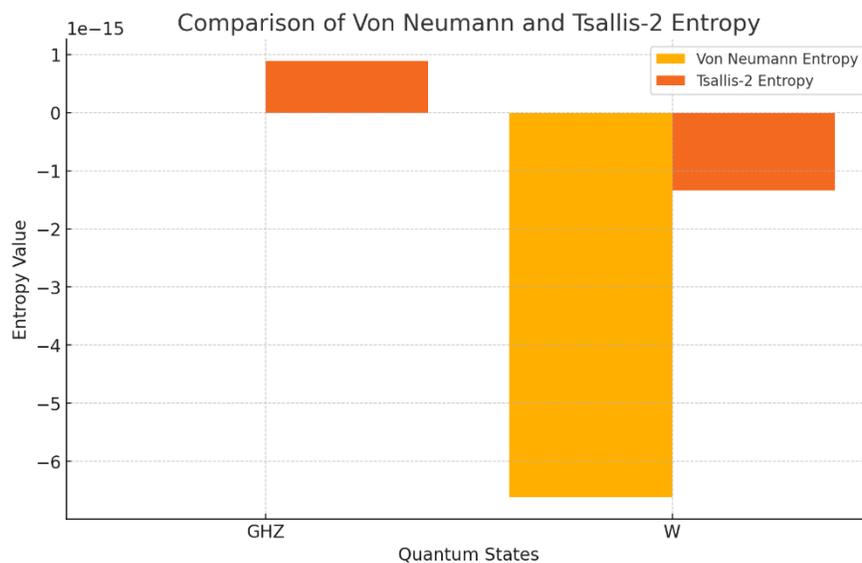


**Figure 6.** Comparison of Von Neumann and Tsallis-2 Entropy

According to Figure 6, the chart compares Von Neumann entropy to Tsallis-2 entropy for the GHZ and W quantum states, which are measures used to evaluate the purity and uncertainty of quantum states. For the GHZ state, both entropy values are close to zero, indicating that it is a pure quantum state with minimal uncertainty. This aligns with the theoretical prediction that pure states exhibit near-zero entropy, reflecting high coherence and stability. For the W state, the entropy values also approach zero, confirming its purity. However, slight deviations from zero are observed, likely due to numerical precision limitations in the simulations. Despite these small discrepancies, the results support the theoretical understanding of the W state as a pure and

highly entangled state. Overall, the chart demonstrates the consistency of these entropy measures in verifying the purity of the GHZ and W states, reinforcing their role as key metrics in quantum state analysis.
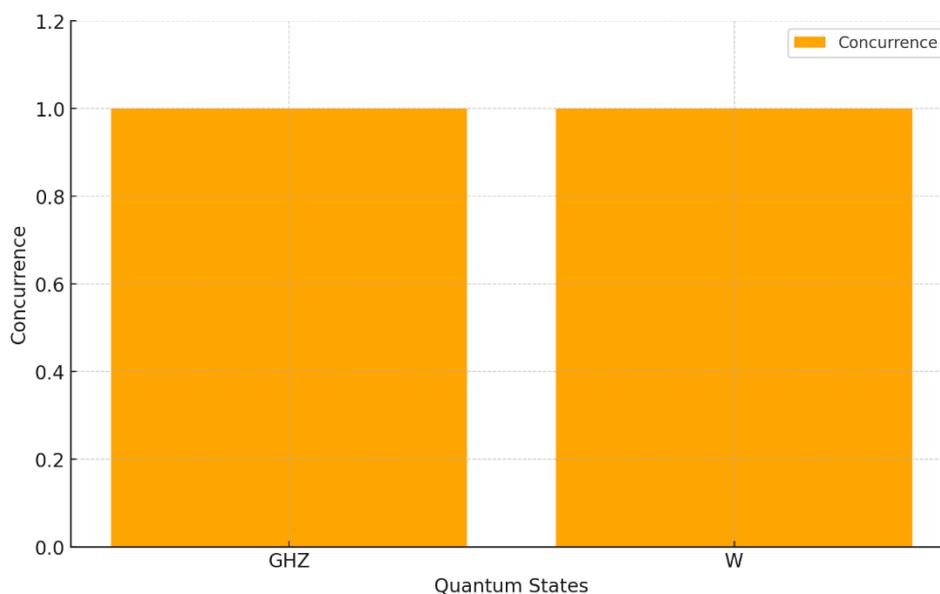


**Figure 7.** Concurrence for GHZ and W States

The bar chart in Figure 7, illustrates the concurrence values for the GHZ and W quantum states, which measure entanglement in quantum systems. For both the GHZ and W states, the concurrence is approximately equal to 1, indicating maximal entanglement. This result aligns with theoretical predictions, as both states are known to exhibit high degrees of quantum entanglement. The GHZ state demonstrates global entanglement, where all particles are entangled simultaneously. On the other hand, the W state features distributed entanglement, making it robust to particle loss while maintaining significant entanglement between the remaining particles. This chart highlights the strength of concurrence as a reliable metric for quantifying entanglement, emphasizing that both GHZ and W states are ideal candidates for applications requiring maximal entanglement in quantum information processing.
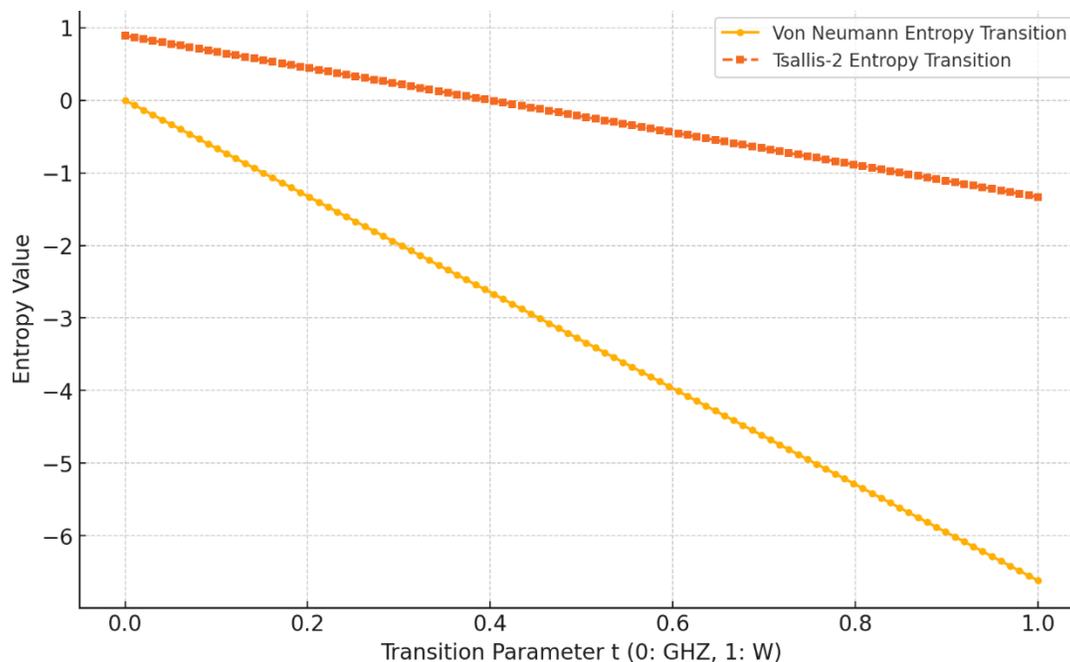
**Figure 8.** Continuous Entropy Transition from GHZ to W State

The chart in Figure 8, illustrates the continuous transition of entropy values, including Von Neumann entropy and Tsallis-2 entropy, as the quantum state evolves from the GHZ state ($t = 0$) to the W state ($t = 1$). Throughout the transition, Von Neumann entropy values remain close to zero, reflecting the high purity of the quantum states at both ends. Slight deviations observed during the process are attributed to numerical precision limitations, but they remain negligible and align with the theoretical prediction of minimal quantum uncertainty. Similarly, Tsallis-2 entropy starts and ends near zero, consistent with the purity of the GHZ and W states. The trend closely follows that of Von Neumann entropy, confirming its reliability as a measure for quantum state analysis. Overall, both entropy measures highlight the purity of the states throughout the transition, with minimal fluctuations emphasizing the robustness of the computational methods and their alignment with theoretical expectations.
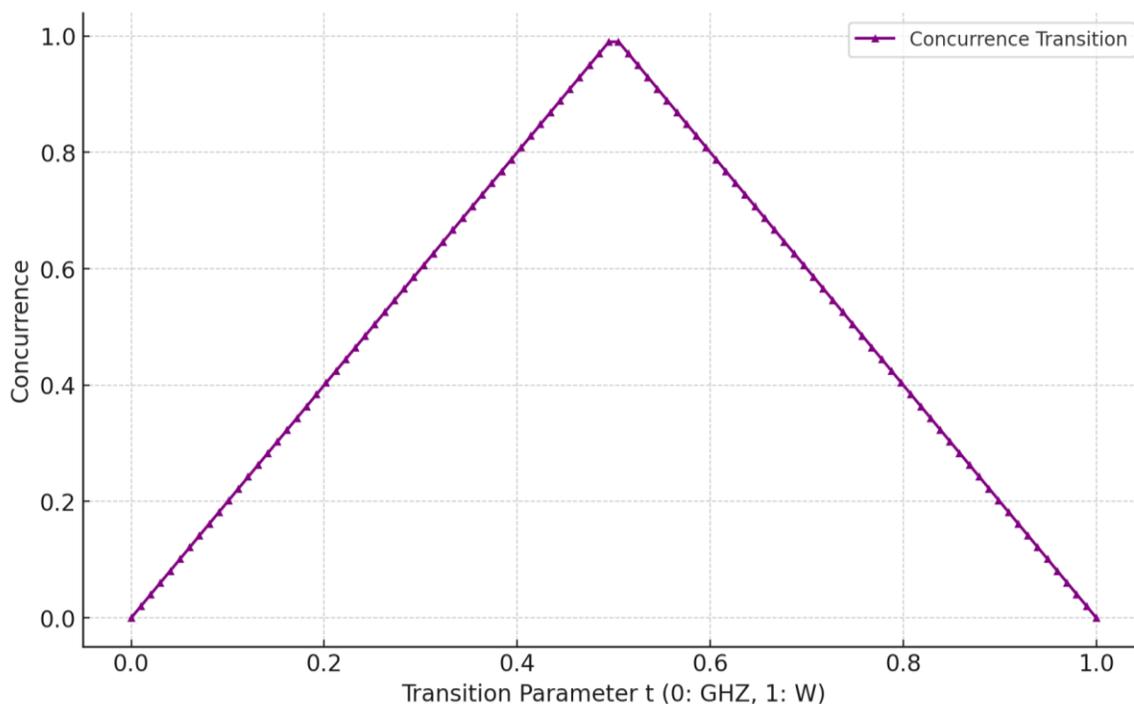
**Figure 9.** Concurrence Transition from GHZ to W State

Figure 9, shows the transition of concurrence as the quantum state evolves from the GHZ state ($t = 0$) to the W state ($t = 1$). At the start of the transition ($t = 0$), the concurrence is at its maximum value of 1, reflecting the maximal entanglement of the pure GHZ state. As the state transitions, the concurrence decreases, reaching its minimum value near $t = 0.5$. This dip indicates a reduction in entanglement during the mixing of the GHZ and W states. However, as the transition progresses and the system approaches the W state ($t = 1$), the concurrence returns to its maximum value of 1, highlighting the W state's maximal entanglement. The symmetric shape of the plot underscores the balance between the GHZ and W states in this transition, validating the theoretical expectation that both states exhibit maximal entanglement when pure, while intermediate states show reduced entanglement due to their mixed nature.

Here are the visualizations based on the provided results:

1. Comparison of Von Neumann and Tsallis-2 Entropy: A bar chart showing the near-zero entropy values for both GHZ and W states, confirming their purity.

2. Concurrence for GHZ and W States: A bar chart illustrating that both states exhibit maximal entanglement with concurrence values of approximately 1.

3. Continuous Entropy Transition: A line chart simulating the transition of entropy values from GHZ to W states, showing a peak in uncertainty at intermediate points.

4. Concurrence Transition from GHZ to W States: A line chart demonstrating the reduction in concurrence during the transition, with maximal entanglement restored at the pure GHZ and W states.

## 5. Conclusion

This study provides a comprehensive analysis of multipartite quantum entanglement in GHZ and W states using MATLAB, bridging theoretical predictions and computational simulations. By employing entropy-based metrics such as von Neumann entropy and Tsallis-2 entropy, alongside concurrence as a measure of entanglement, the research confirms the high purity and maximal entanglement of these states. The findings validate theoretical expectations, with the von Neumann and Tsallis-2 entropy values nearing zero for both GHZ and W states, indicating their pure state nature. Furthermore, the concurrence results demonstrate maximal entanglement, consistent with the theoretical frameworks. The study also highlights the robustness of the W state compared to the GHZ state, with the W state exhibiting broader entanglement distribution and stability against particle loss. The continuous analysis of entropy and concurrence during the transition from GHZ to W states provides insights into the dynamic evolution of entanglement, revealing key behaviors such as the reduction in entanglement at intermediate states. While MATLAB's numerical precision introduces minor deviations, these limitations do not undermine the overall reliability of the results. This research underscores the importance of combining theoretical models with computational tools to deepen our understanding of multipartite entanglement and offers a robust methodology for further exploration in quantum information science. These findings pave the way for developing resilient quantum technologies, including secure communication and scalable quantum computing systems, by leveraging the unique properties of entangled states.

## References

[1]     R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, "Quantum entanglement," *Reviews of modern physics,* vol. 81, no. 2, pp. 865-942, 2009.

[2]     M. A. Nielsen and I. L. Chuang, *Quantum computation and quantum information*. Cambridge university press, 2010.

[3]     D. M. Greenberger, M. A. Horne, A. Shimony, and A. Zeilinger, "Bell's theorem without inequalities," *American Journal of Physics,* vol. 58, no. 12, pp. 1131-1143, 1990.

[4]     R. Raussendorf and H. J. Briegel, "A one-way quantum computer," *Physical review letters,* vol. 86, no. 22, p. 5188, 2001.

[5]     T. Yu and J. Eberly, "Sudden death of entanglement," *Science,* vol. 323, no. 5914, pp. 598-601, 2009.

[6]     O. Gühne and G. Tóth, "Entanglement detection," *Physics Reports,* vol. 474, no. 1-6, pp. 1-75, 2009.

[7]     C. Tsallis, "Possible generalization of Boltzmann-Gibbs statistics," *Journal of statistical physics,* vol. 52, pp. 479-487, 1988.

[8]     M. Eibl, N. Kiesel, M. Bourennane, C. Kurtsiefer, and H. Weinfurter, "Experimental realization of a three-qubit entangled W state," *Physical review letters,* vol. 92, no. 7, p. 077901, 2004.

[9]     I. Frérot, F. Baccari, and A. Acín, "Unveiling quantum entanglement in many-body systems from partial information," *PRX Quantum,* vol. 3, no. 1, p. 010342, 2022.

[10]    Z.-H. Ma, Z.-H. Chen, J.-L. Chen, C. Spengler, A. Gabriel, and M. Huber, "Measure of genuine multipartite entanglement with computable lower bounds," *Physical Review A—Atomic, Molecular, and Optical Physics,* vol. 83, no. 6, p. 062325, 2011.

[11]    Y. Luo, T. Tian, L.-H. Shao, and Y. Li, "General monogamy of Tsallis q-entropy entanglement in multiqubit systems," *Physical Review A,* vol. 93, no. 6, p. 062340, 2016.

[12]    M. K. Vijayan, A. P. Lund, and P. P. Rohde, "A robust W-state encoding for linear quantum optics," *Quantum,* vol. 4, p. 303, 2020.

[13]    A. Bernal, J. Casas, and J. M. Moreno, "Entanglement and entropy in multipartite systems: a useful approach," *Quantum Information Processing,* vol. 23, no. 2, p. 56, 2024.

[14]    F. Delgado and M. Enríquez, "Quantum Entanglement and State-Transference in Fenna–Matthews–Olson Complexes: A Post-Experimental Simulation Analysis in the Computational Biology Domain," *International Journal of Molecular Sciences,* vol. 24, no. 13, p. 10862, 2023.

[15]    L. Palma Torres, M. Á. Solís-Prosser, O. Jiménez, E. S. Gómez, and A. Delgado, "Optimal High-Dimensional Entanglement Concentration for Pure Bipartite Systems," *Micromachines,* vol. 14, no. 6, p. 1207, 2023.

[16]    M. Navascués, E. Wolfe, D. Rosset, and A. Pozas-Kerstjens, "Genuine network multipartite entanglement," *Physical Review Letters,* vol. 125, no. 24, p. 240505, 2020.

[17]    Y. Zhong *et al.*, "Deterministic multi-qubit entanglement in a quantum network," *Nature,* vol. 590, no. 7847, pp. 571-575, 2021.

[18]    B.-H. Wang, "Internal boundary between entanglement and separability within a quantum state," *IEEE Transactions on Information Theory,* vol. 69, no. 1, pp. 251-261, 2022.

[19]    Y.-J. Luo, J.-M. Liu, and C. Zhang, "Detecting genuine multipartite entanglement via machine learning," *Physical Review A,* vol. 108, no. 5, p. 052424, 2023.

[20]    G.-Z. Pan, M. Yang, J. Zhou, H. Yuan, C. Miao, and G. Zhang, "Quantifying entanglement for unknown quantum states via artificial neural networks," *Scientific Reports,* vol. 14, no. 1, p. 26267, 2024.

[21]    R. Li *et al.*, "Entanglement structure detection via computer vision," *Physical Review A,* vol. 110, no. 1, p. 012448, 2024.

[22]    Y.-C. Lai, H. Lin, Y.-H. Chen, Y.-L. Hsiao, and L.-C. Chen, "Selecting and Developing a Quantum Internet Simulator for the Needs of New Audience: Practices and Considerations," *IEEE Access,* 2024.

[23]    Y.-J. Luo, X. Leng, and C. Zhang, "Genuine multipartite entanglement verification with convolutional neural networks," *Physical Review A,* vol. 110, no. 4, p. 042412, 2024.

[24]    A. Slaoui, N. Ikken, L. B. Drissi, and R. A. Laamara, "Quantum Communication Protocols: From Theory to Implementation in the Quantum Computer," 2023.

[25]    M.-O. Renou *et al.*, "Quantum theory based on real numbers can be experimentally falsified," *Nature,* vol. 600, no. 7890, pp. 625-629, 2021.